



Politecnico  
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# PURE AND APPLIED MATHEMATICS

**Towards atomtronic: mathematics of ultracold gases**



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<b>Context of the research activity</b>	<p>In the last years an emerging technology aims at exploiting circuits in which currents consist of matter flows instead of electron flows. Here matter typically means ultracold gases or Bose-Einstein condensates. This technological challenge entails many mathematical issues involving calculus of variations, spectral theory, nonlinear dynamics to be developed on nonstandard structures like graphs and hybrids.</p>
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	<p>Atomtronics is the name given to every technology that exploits circuit-like devices using matter instead of electrons. By “matter” one here understands cold gases of bosons, condensed according to the discovery reached by Bose and Einstein one hundred years ago. In fact, atomtronics puts together two late ideas by Einstein: first, the fact that, when driven to a low enough temperature, a gas of identical bosons undergoes a phase transition. This process is called “Bose-Einstein condensation”. In the new phase, the whole gas behaves as a unique, giant, quantum particle that obeys a nonlinear evolutionary law. The second idea is the quantum entanglement, namely the persistent correlation between quantum systems that interacted in the past, regardless of their spatial and time separation. From the mathematical point of view, the framework is given by either a many-body linear Schroedinger equation, when one considers all degrees of freedom of the gas, or a one-body nonlinear Schroedinger equation, called in this context Gross-Pitaevskii equation, that rules the time evolution of the condensates. The natural environment where to define such equation is a hybrid structure, i.e. a structure made of pieces of different spatial dimensions. In order to describe the matter current on a hybrid device one has to choose suitable boundary</p>
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## Objectives

conditions that guarantees the fundamental conservation laws (mass and energy) and that is physically significant. To this aim, it will be crucial to interact with physicists working on junctions, like Prof. Andrea Trombettoni in SISSA-Trieste. The final aim of the research is to isolate approximate solutions describing the conduction in atomtronic device, estimate the error in the approximation, specifying thus the range of validity of the resulting specific regimes. A collaboration has been established with Prof. Benjamin Schlein, one of the leader expert in the field on Many-Body Quantum systems, currently Professor at the University of Zuerich. Another important connection is with Prof. Ugo Boscain, worldwide known as an expert in Quantum Control, currently Directeur de Recherche for the CNRS at the Université Sorbonne in Paris. We are planning also contacts with leader enterprises in the field of Quantum Technology like Italtel. Technically, an important step is the characterization of the quantum states of condensed cold gases in hybrid structures that model the atomtronic devices. Then, the dynamics of such gases in the hybrid structures will be studied. This will lead to new functional spaces to be studied.

## Skills and competencies for the development of the activity

The candidate should have some background in functional analysis, especially in calculus of variations and spectral theory. An elementary knowledge of quantum mechanics could be useful. However, during the first six months the candidate is supposed to learn more about the topics involved in the project.