

PhD in Pure and Applied Mathematics

Research Title: Multivariate models for option pricing and hedging.

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Supervisor	Paolo Brandimarte paolo.brandimarte@polito.it Patrizia Semeraro patrizia.semeraro@polito.it
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Contact	https://www.disma.polito.it/la_ricerca/gruppi/probabilita_e_applicazioni
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Context of the research activity	<p>A derivative can be defined as a financial instrument whose value depends on (or derives from) the values of other, more basic, underlying variables. Derivatives are actively traded on many exchanges throughout the world, and there is also a huge over-the-counter (OTC) market. They are used for hedging risk, speculation or arbitrage. It is therefore fundamental to understand how derivatives work and how they are used by both financial institutions and non-financial firms.</p> <p>In this backdrop, two fundamental issues to be addressed by financial institutions and non-financial firms are derivative pricing -under a no arbitrage assumptions- and risk management. The latter includes hedging the risk of writing options and using derivatives to hedge risk. Derivative pricing is also essential for non-financial institutions to check the sensibility of OTC prices, which are not traded on liquid markets.</p> <p>Derivatives may depend on several underlying variables (not necessarily limited to traded assets), and it is important to be able to deal with multivariate and realistic models, which must be properly calibrated before being used for pricing and risk management.</p>
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	<p>Multi-asset derivative pricing is still an active field of research in financial modeling, calling for multivariate stochastic models that reproduce well-known stylized facts such as skewness and excess kurtosis of marginal return distributions. These models incorporate jumps and lead to incomplete markets, where one cannot in general form a perfect hedge of a given position. Furthermore, some risk factors (like volume risk) cannot be hedged easily, leading to residual risk.</p> <p>This leaves us with the problem of the determination of the optimal strategy for hedging a financial position, given the set of assets at the hedger's disposal. The optimal hedge corresponds to the best possible elimination of risk (as measured by suitable risk functionals) in a financial position making use of the instruments available for this purpose.</p>
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Objectives	<p>The objectives of this research project are to build stochastic models for financial assets, develop calibration procedures and simulation methods. Simulations are necessary to price derivatives if analytical pricing formulas are not available, and they are also necessary to generate future scenarios for out-of-sample evaluation.</p> <p>A class of models widely used in finance are Markov processes. Among them, Lévy processes, which are characterized by independent and time-homogeneous increments, are widely used because of their analytical tractability and good fit on financial data. These processes are recently used to build multivariate models able to incorporate linear and non-linear dependences among assets. In fact, both linear and non-linear dependence may affect the price of a multi-asset derivative or the risk associated to a financial position.</p> <p>One of the objectives of the present project is to build and characterize multivariate stochastic processes able to reproduce well-known stylized facts and with a flexible dependence structure. A way to build multivariate stochastic models in finance is to subordinate a Lévy process or, more in general of a Markov process, e.g., Ornstein-Uhlenbeck process. Subordination has the nice economic interpretation of a change of time. The underlying idea is that the time runs fast when there are a lot of order, while it slows down when trade is stale. If the change of time is a Lévy subordinator, it introduces jumps in the model leading an incomplete market.</p> <p>Once the model has been built the objective becomes to calibrate the models on real data, both under the historical and under the risk neutral measures and evaluate its fit properties.</p>
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	<p>After characterizing the stochastic model for the underlying risk factors, we must price the derivative. In the case of a European-style derivative, we may just need efficient implementation of Monte Carlo procedures. However, stochastic optimization models are needed to deal with early exercise features. Furthermore, stochastic optimization models are required to find the optimal hedge or the optimal risk management strategy under an incomplete market. To this aim, we must introduce proper risk measures, careful scenario generation, and suitable numerical optimization methods.</p> <p>Another critical issue is related to model risk. The more sophisticated the model is, the more prone to misspecification and calibration errors. Hence, we have to address issues in distributional ambiguity and the robustness of the risk management strategy must be built into the model and carefully analyzed out of sample.</p>
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Skills and competencies for the development of the activity	<p>The required competencies to carry out the research work include:</p> <ul style="list-style-type: none">• Theory of stochastic processes, as well as some notions about stochastic calculus.• Numerical methods and ability to implement numerical procedures in MATLAB and Python, possibly using parallel computing and object-oriented design.• Dynamic optimization under uncertainty, including stochastic programming with recourse, robust optimization, and stochastic dynamic programming (including, but not limited to reinforcement learning).• The practical knowledge of financial engineering topics like structure of derivatives, pricing, hedging and risk management are also useful.
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